Radiative Properties of Rough Surfaces

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Most studies on surface roughness effects are based on geometrical optics. This analytical investigation treats the surface roughness effects from wave optics considerations, thus accounting for the interference of waves. The rough surface is sinusoidal, infinite in extent and it is irradiated by plane monochromatic waves. The scattering of electromagnetic waves by the rough surface is determined by the direct solution of Maxwell's equations instead of the commonly used Huygens-Kirchhoff approximation (Beckmann's reflectance). Consequently, the solution is not restricted to small wavelength to roughness ratios; furthermore, polarization effects can be accounted for. The results indicate that the transmittance of the rough surface exhibits a maximum near the point where the wavelength of the surface roughness is the same as the wavelength of the radiation. The interval around this point is called transition region because the surface changes from specular to diffuse. The radiative properties are influenced by both the pitch and amplitude of the roughness. The results approach Fresnel's law when the pitch and amplitude of the surface become small compared to the wavelength of the radiation. Similarly, the surfaces become specular even at larger pitch if the amplitude of the roughness is small. It is also shown that Beckmann's reflectance relation is the limiting case of the reflectance obtained by the direct solution of Maxwell's equations. In addition, numerical results are presented on the effective properties of rough dielectric plates with various refractive indices and optical thicknesses.

Nomenclature

A = coefficient in the vector equation
a = amplitude of the surface roughness
B = coefficient in the vector equation
$\mathbf{B}$ = magnetic flux density
B_ν = Planck function
b = parameter in Beckmann's relation
C = coefficient in the vector equation
D = coefficient in the vector equation
$\mathbf{D}$ = electric flux density
d = pitch (or wavelength) of the roughness
db = decibel
$\mathbf{E}$ = electric field amplitude
$\dot{\mathbf{E}}$ = electric field vector
f = aspect ratio or shape factor
$\mathbf{H}$ = magnetic field amplitude
$\dot{\mathbf{H}}$ = magnetic field vector
h = surface roughness in rms
I = intensity of radiation
J = Bessel function of the first kind
j = $[-1]^{1/2}$
k = wave number
L = thickness of the plate
m = index for diffracted modes, integer
N = index in the wave equation
n = refractive index
$\mathbf{n}$ = surface unit normal vector
P = coefficient in the vector equation
Q = coefficient in the vector equation
r = reflectance
$\mathbf{S}$ = Poynting vector
s = path in the transport equation
t = transmittance
$\mathbf{u}$ = unit normal vector
x = coordinate taken normal to the generators of the rough surface
y = coordinate parallel to the generators of the rough surface
z = coordinate normal to the central plane of the rough surface
α = absorption index
α_m = directional cosine of reflected waves (with subscript m)
β_m = product of directional cosine and refractive index for refracted waves
ϵ_ν = effective emittance of plate
θ = diffraction angle or angle of incidence
λ = wavelength of radiation in vacuum
μ = directional cosine in the transport equation
ρ_ν = effective reflectance of plate
τ = optical thickness
τ_ν = effective transmittance of the plate (with subscript ν)

Subscripts

c = critical angle
i = subscript in the vector equation for the wave amplitudes
m = order of diffracted modes and directional cosines
x = x direction
y = y direction
z = z direction
ν = frequency

Superscripts

i = incident
r = reflected
t = refracted or transmitted
$'$ = rough interface or plate rough on one side
$''$ = plate rough on both sides

Introduction

PROPAGATION of electromagnetic waves through the interface of different media has been the subject of numerous investigations in the past. Solution methods are discussed in the literature of electromagnetic theory and optics and the results are well known as some of the basic laws of wave propagation such as Fresnel's law of reflection and Snell's law. These laws have great practical significance in all wavelength bands insofar as they predict the behavior of energy reflected and transmitted at the interface of the two media. In the derivation of the laws it is assumed in general that the

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interface of the media is perfectly smooth; i.e., diffraction effects due to surface irregularities are neglected. Most of the technical surfaces, however, are nonsmooth; even the best polished optical glass exhibits scratches and grooves on the order of a few Angstroms deep. Thin films are granular in surface structure and wavy due to imperfections in the substrate material. Diffraction gratings for spectroscopy are man-made rough surfaces. The surface of the sea is rough due to the waves. It can be seen that the concept "smoothness" is relative; it depends on the size of the roughness, on the wavelength of the radiation and on the angle of incidence of the radiation. Surfaces can be considered smooth if the roughness is much smaller than the wavelength.

The theory of gratings is more than half a century old, starting with the development of the dynamical theory of gratings by Rayleigh[1] in 1907. Later in 1938, Fano[2] extended Rayleigh's method. The theory has been the subject of intensive investigation in recent years, by Stroke,[3] Bousquet,[4] Wirgin,[5] and especially Pavageau[6] among others. Another approach, although less useful for the present problem is the Kirchhoff method, described in detail by Beckmann.[7] Grating theories predict reflection and transmission for rough surfaces, thus replacing or modifying Fresnel's law. The objective of the present study is to assess the effect of surface roughness on radiative surface properties. Furthermore, the combined effect of surface roughness and bulk properties of materials are discussed.

In the present study, first the radiative properties of rough surfaces with sinusoidal contour are determined by solving Maxwell's equations. Randomly rough surfaces are also discussed and test results are compared with analytical predictions. Finally, the surface properties are used to obtain apparent radiative properties of dielectric plates with rough interfaces and with various optical thicknesses. These results are compared with those obtained by Francis and Love[8] for smooth plates.

Analysis

1. Formulation of the Problem

The problem considered in this study is the propagation of radiation in the vicinity of a rough surface (Fig. 1). The roughness is described by a sinusoidal contour curve and characterized by the roughness amplitude a and the roughness wavelength (or pitch) of d. The grooves formed by the sinusoidal contour are infinite in extent and run parallel to the y-axis. The upper plane is vacuum, the lower one is a homogeneous material with a given refraction index n (larger than unity) and with small absorption index α.

The rough surface is irradiated by infinite monochromatic plane waves. Waves impinging at the interface give rise to a scattered wave pattern on both sides of the interface. Part of the incident energy is reflected in the specular direction. Another part is transmitted through the interface in the direction determined by Snell's law of refraction. The remainder of the energy is scattered around the specular (or zeroth order) beams. The whole pattern is a diffraction field, where the directions of higher order beams can be determined by the grating equation, a simple trignometric relation defining discrete direction in which path differences of radiation originated from the same source results in constructive interference. This interference effect is most significant if the characteristic dimension of the roughness (pitch, d) is the same order of magnitude as the wavelength of the radiation.

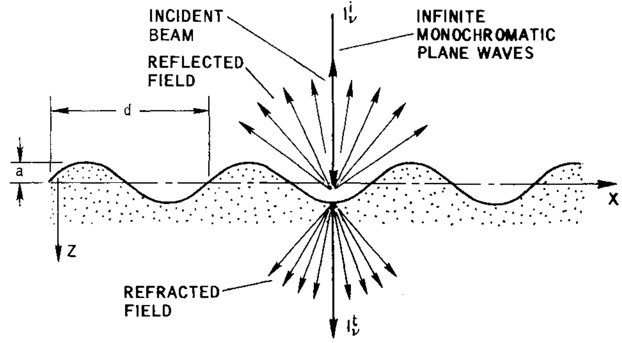

Fig. 1 Propagation of radiation through an infinite rough plate.

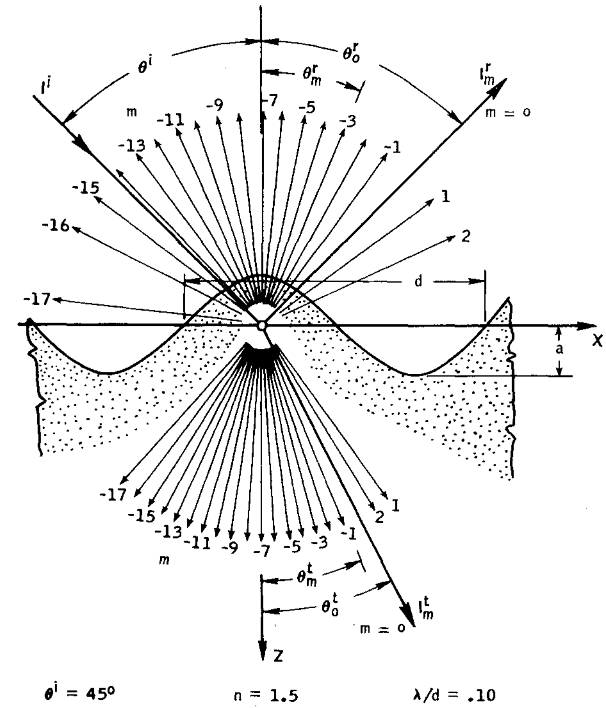

Fig. 2 The diffracted field.

Figure 2 illustrates the diffracted field for an angle of incidence of 45°.

In order to obtain the radiative surface properties, the diffracted field has to be determined by solution of the electromagnetic field equations. For optically smooth surfaces, where only the zeroth order field components appear (specular behavior) the surface properties are expressed in Fresnel's equations.

2. General Considerations of Properties of Rough Surfaces

Qualitative description of the effect of surface roughness on the radiative properties, such as reflectance, emittance and transmittance was given by Hottel.[9] According to Hottel the roughness can be characterized by the amplitude, a of the surface contour or an equivalent expression in terms of the root mean square of the roughness. The ratio a/λ can be used as a criterion to define the effect of roughness on the radiative properties. Based on this ratio, the following conditions can be considered a) $a/\lambda \ll 1$. The reflectance is specular. Fresnel's law is applicable up to a certain a/λ ratio, b) $a/\lambda \cong 1$. The roughness is the same order of magnitude as the wavelength. Diffraction effects become important: c) $a/\lambda \gg 1$. The law of geometrical optics is applicable. Fresnel's relation can be used to predict reflectance on smooth portions of large irregularities. Diffraction effects may cause perturbations in the properties.

Quantitative evaluation of the effect of surface roughness on the radiative properties can be accomplished according to the laws of electromagnetic wave propagation by using the grating theory. Change of specular reflection into diffuse scattering was explained by Rayleigh. When two rays are specularly

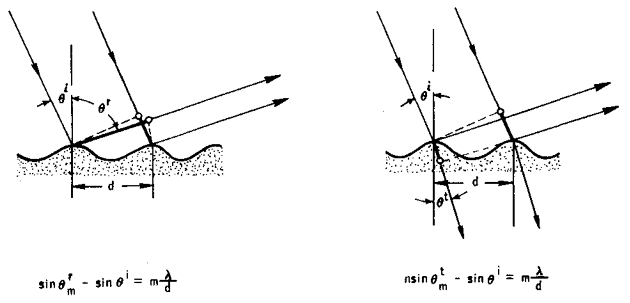

Fig. 3 Geometry for the grating equation.

reflected from rough surfaces, it may occur that the reflected beams experience a phase shift of π, due to the roughness, and the rays cancel. Consequently, there is no outflow of energy and it has to be redistributed in other directions, resulting in a spectrum of rays (Fig. 1). The direction of rays are determined by the grating equation, showing that not only the amplitude of the roughness, but also the wavelength of the roughness is an important parameter in determining the surface properties.

3. Grating Equation

Figure 3 illustrates the interference path of rays for reflection and refraction. By trigonometric relations it can be seen that constructive interference occurs if

$$n \sin\theta_m{}^t = \sin\theta_m{}^r = \sin\theta^i + m\frac{\lambda}{d} \tag{1}$$

where θ^i, θ^r and θ^t are the angle of incidence, reflection and transmission respectively, and m is an integer $m = 0, \pm1, \pm2, \pm3, \pm4, \ldots$

4. Scattering Theories—Kirchhoff's Method

This method consists in approximating the boundary conditions on the surface and then determining the diffracted field by using the Helmholtz integral. Although the method is not directly applicable for the solution of the present problem due to limitations which will be discussed later, it provides the upper bound of the solution for the scattered field obtained by Rayleigh's method for the limiting case when the refractive index approaches infinity. Detailed description of Kirchhoff's method is given by Beckmann.[7]

For a semi-infinite sinusoidally rough surface whose contour is defined by the relation

$$z = a \cos 2\pi x/d \tag{2}$$

Beckmann gives the amplitude reflection coefficient. Based on his relation, the power reflection coefficient can be written in terms of the incident angle, the reflected spectrum and Bessel functions of the first kind as

$$r'(\theta^i,\theta_m{}^r) = [(-j)^m J_m(b)\{1 + \cos(\theta^i + \theta_m{}^r)\}/(\cos^2\theta^i + \cos^2\theta^i \cos\theta_m{}^r)]^2 \tag{3}$$

where

$$m = 0, \pm1, \pm2, \pm3, \ldots$$

$$b = ka\,(\cos\theta^i + \cos\theta_m{}^r)$$

and $k = 2\pi/\lambda$ is the wave number. The reflected spectrum, $\theta_m{}^r$ is determined by the grating equation.

Parameter b can also be expressed as

$$b = 2\pi a/\lambda\,(\cos\theta^i + \cos\theta_m{}^r) \tag{4}$$

Equations (1, 3, and 4) show that the interaction of radiation and the rough surface can be characterized by two dimensionless quantities a/λ and λ/d in addition to the angle of incidence θ^i. For normal incidence, $\theta^i = 0$ and Eq. (3) yields

$$r'(\theta_m{}^r) = [(-j)^m J_m(b)]^2 \tag{5}$$

The reflectance related to the zeroth order (specular) beam for normal incidence is

$$r' = J_0{}^2(4\pi a/\lambda) \tag{6}$$

Graphical representation of the preceding equation is given in Fig. 4. It should be noted that in Eq. (6) the reflectance is independent of the pitch of the roughness contour. The scattered spectrum however is a function of the pitch, expressed by Eq. (5) which in turn depends on the grating Eq. (1) and parameter b. Equation (1) indicates that for normal incidence the first order ($m = \pm1$) reflected beams appear as soon as the pitch of the roughness, d exceeds the wavelength of the radiation λ. Increasing d results in more diffused reflected field.

The major advantage of the Kirchhoff method is its simplicity; the reflection coefficient can be expressed in a closed form relation. Some of the disadvantages of the method are that it gives only the reflected field without giving any indication of the refracted field. Furthermore it is valid only for $d/\lambda \gg 1$. The slope of the roughness which could be taken proportional to a/d, should also be small. In addition, polarization effects cannot be accounted for.

5. Scattering Theories—Rayleigh's method

The electromagnetic field above and in the dielectric are described by Maxwell's equations

$$\nabla \times \mathbf{E} = -\partial\mathbf{B}/\partial t, \quad \nabla \times \mathbf{H} = \partial\mathbf{D}/\partial t, \quad \nabla\cdot\mathbf{B} = 0, \quad \nabla\cdot\mathbf{D} = 0 \tag{7}$$

These equations are solved by Rayleigh's method, which postulates the diffracted field as the sum of plane waves and then determines the unknown amplitudes by satisfying the boundary conditions at the interface. Both the reflected and refracted fields are represented and the solutions are valid for any values of the parameters a/λ and λ/d. There are no constraints relative to the properties of the media, although the original work by Rayleigh was based on the assumption that the conductivity of the material with rough surface is infinite. Recent diffraction theories (Bousquet, Pavageau), however, allow for both lossless and lossy media, i.e., for real and complex refractive indices. The only probable disadvantage of Rayleigh's method is the relative complexity of the solution, which might not be too serious in view of the availability of high speed digital computers.

In the following, the Rayleigh method will be described for waves incident normally on a semi-infinite sinusoidal rough surface. The method of solution is based on investigations

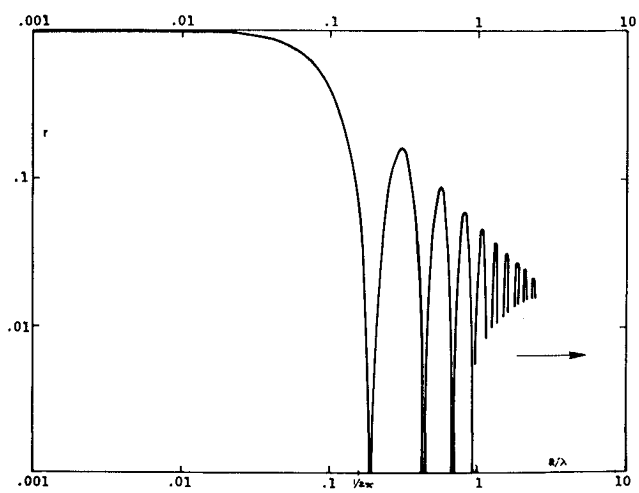

Fig. 4 Beckmann's normal reflectance.

by Bousquet and Pavageau. The medium is a dielectric with low loss so that the refractive index can be considered as real. Figure 5 illustrates the conditions. Infinite monochromatic plane electromagnetic waves (TEM) propagate in the positive z direction. The surface is sinusoidal and cylindrical, i.e., the grooves are infinitely long in the y direction. The incident field is polarized the way that the electric vector is parallel to the axis. Upon reaching the surface, the incident waves give rise to a spectrum of plane waves (Fig. 2). The direction of propagation of the diffracted waves is determined by the grating equation and the amplitudes of the waves are derived based on the solution of the wave equation (or the Helmholtz equation) which in turn can be derived from Maxwell's equations.

The incident electric field is described by the relation

$$\dot{\mathbf{E}}^i = \mathbf{E}^i \exp(-jkz) \quad (8)$$

where $k = 2\pi/\lambda$ is the wave number. $\dot{\mathbf{E}}^i$ is the instantaneous incident electric vector and $\mathbf{E}^i$ is the amplitude of the incident radiation. The incident field upon reaching the rough interface gives rise to a spectrum of reflected plane waves which can be described by the equation

$$\dot{\mathbf{E}}_m{}^r = \mathbf{E}_m{}^r \exp[-jk(x \sin\theta_m{}^r - z \cos\theta_m{}^r)] \quad (9)$$

Similarly, a spectrum of refracted plane waves is being generated in the interior of the dielectric across the interface, which can be written as

$$\dot{\mathbf{E}}_m{}^t = \mathbf{E}_m{}^t \exp[-jkn(x \sin\theta_m{}^t + z \cos\theta_m{}^t)] \quad (10)$$

Index m is identical to the one used in the grating equation

$$\sin\theta_m{}^r = n \sin\theta_m{}^t = m \frac{\lambda}{d}$$

where

$$m = 0, \pm 1, \pm 2, \pm 3, \ldots \quad (11)$$

In order to obtain the reflection and transmission coefficients at the interface, amplitudes $\mathbf{E}_m{}^r$ and $\mathbf{E}_m{}^t$ have to be determined by satisfying the wave equation. The unknown amplitudes for the diffracted field have to be determined based on the boundary conditions, that they also satisfy the wave equation, which can be stated in terms of the electric field as

$$\nabla^2\dot{\mathbf{E}} + k^2N^2\dot{\mathbf{E}} = 0 \quad (12)$$

where $N = 1$ for free space and $N = n$ for the dielectric. The nature of the diffracted waves can be defined based on the directional cosines of the waves

$$\alpha_m = \cos\theta_m{}^r = [1 - m^2(\lambda/d)2]^{1/2} \quad (13)$$

$$\beta_m = n \cos\theta_m{}^t = [n^2 - m^2(\lambda/d)^2]^{1/2} \quad (14)$$

where $m = 0$, specular case; $\alpha_m{}^2 > 0, \beta_m{}^2 > 0$ traveling (real) waves; $\alpha_m{}^2 < 0, \beta_m{}^2 < 0$ evanescent (exponentially dampened) waves; $\alpha_m{}^2 = 0, \beta_m{}^2 = 0$ no field. Depending on the conditions, the reflected and transmitted waves of the same order can be evanescent and real respectively at the same time. To find the solution of the problem, the boundary conditions have to be defined for the diffracted field, based on the continuity of the tangential components of the electric and magnetic fields at the interface.

$$\mathbf{n} \times (\dot{\mathbf{E}}^i + \dot{\mathbf{E}}^r - \dot{\mathbf{E}}^t) = 0 \quad (15)$$

$$\mathbf{n} \times (\dot{\mathbf{H}}^i + \dot{\mathbf{H}}^r + \dot{\mathbf{H}}^t) = 0 \quad (16)$$

where $\mathbf{n}$ is the unit normal vector of the rough surface, which can be written for sinusoidal surfaces as

$$\mathbf{n} = [\mathbf{u}_x(2\pi)/(d)a \sin 2\pi(x/d) + \mathbf{u}_z]/[1 + 4\pi^2/d^2(a^2) \sin^2 2\pi(x/d)]^{1/2} \quad (17)$$

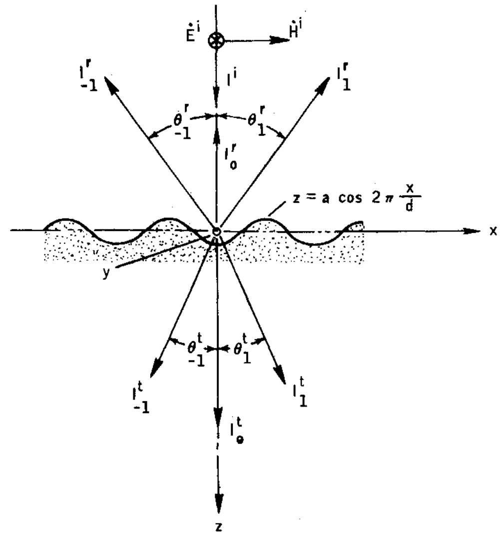

Fig. 5 Propagation of waves at a rough surface.

where $\mathbf{u}_x$, $\mathbf{u}_y$ and $\mathbf{u}_z$ are the unit vectors in the directions of x, y and z axes, respectively.

After performing the operations indicated in Eqs. (15) and (16) the boundary conditions can be written as

Electric field

$$\dot{E}^i + \sum_{m=-\infty}^{+\infty} \dot{E}_m{}^r = \sum_{m=-\infty}^{+\infty} \dot{E}_m{}^t \quad (18)$$

Magnetic field

$$\dot{H}^i + \sum_{m=-\infty}^{+\infty} [(\dot{H}_m{}^r)_x - (2\pi a/d) \sin(2\pi x/d)(\dot{H}_m{}^r)_z] = \sum_{m=-\infty}^{+\infty} [(\dot{H}_m{}^t)_x - (2\pi a/d) \sin(2\pi x/d)(\dot{H}_m{}^t)_z] \quad (19)$$

Based on the foregoing equations the unknown amplitudes of the electric field, $E_m{}^r$ and $E_m{}^t$ can be determined. By expressing the electric vector $\dot{\mathbf{E}}_m$ and magnetic vector $\dot{\mathbf{H}}_m$ in terms of the electric vector, $\mathbf{E}_m$ and by developing the trigonometric powers of e in Fourier series in terms of Bessel functions of the first kind, the unknown amplitudes $E_o{}^r$, $E_o{}^t, \ldots, E_m{}^r, E_m{}^t, \ldots$, can be determined from a double infinite system of linear equations by successive approximation. The system of equation can be presented by the following matrix equation for $E^i = 1$:

$$\begin{bmatrix} A_{oo}B_{oo}A_{ol}B_{ol} \ldots A_{om}B_{om} \ldots \\ C_{oo}D_{oo}C_{ol}D_{ol} \ldots C_{om}D_{om} \ldots \\ \ldots \\ \ldots \\ A_{io}B_{io}A_{il}B_{il} \ldots A_{im}B_{im} \ldots \\ C_{io}D_{io}C_{il}D_{il} \ldots C_{im}D_{im} \ldots \\ \ldots \\ \ldots \end{bmatrix} \cdot \begin{bmatrix} E_0{}^r \\ E_0{}^t \\ \cdot \\ \cdot \\ \cdot \\ E_m{}^r \\ E_m{}^t \\ \cdot \end{bmatrix} = \begin{bmatrix} P_0 \\ Q_0 \\ \cdot \\ \cdot \\ \cdot \\ P_i \\ Q_i \\ \cdot \end{bmatrix} \quad (20)$$

where

$$A_{io} = (-1)^{i+1}J_i(ka), \quad B_{io} = J_i(nka)$$

$$C_{io} = (-1)^iJ_i(ka), \quad D_{io} = nJ_i(nka), \quad P_i = J_i(ka)$$

$$Q_i = J_i(ka)$$

and

$$A_{im} = -j^m[J_{m-i}(ka\alpha_m) + (-1)^iJ_{m+i}(ka\alpha_m)]$$

$$B_{im} = j^m[J_{m+i}(ka\beta_m) + (-1)^iJ_{m-i}(ka\beta_m)]$$

$$C_{im} = j_m\{\alpha_m[J_{m-i}(ka\alpha_m) + (-1)^iJ_{m+i}(ka\alpha_m)] + m/2(\lambda/d)^2ka[J_{m-i-1}(ka\alpha_m) + J_{m-i+1}(ka\alpha_m) + (-1)^i(J_{m+i-1}(ka\alpha_m) + J_{m+i+1}(ka\alpha_m))]\}$$

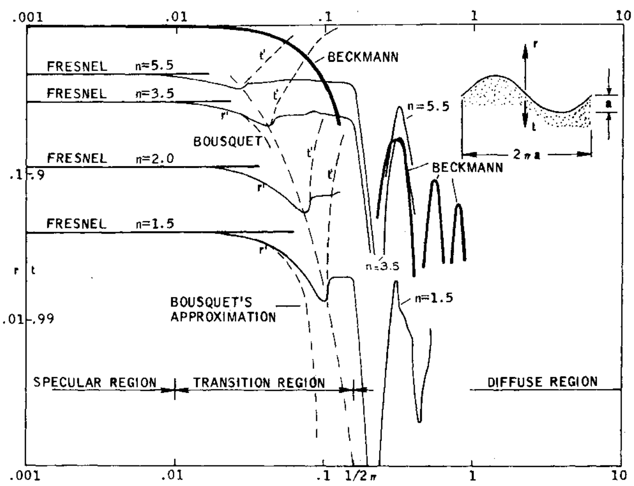

Fig. 6 Surface properties for regular cosine contour.

$$D_{im} = j^{-m}\{\beta_m[J_{m+i}(ka\beta_m) + (-1)^i J_{m-i}(ka\beta_m)] + m/2(\lambda/d)^2 ka[J_{m+i-1}(ka\beta_m) + J_{m+i+1}(ka\beta_m) + (-1)^i(J_{m-i-1}(ka\beta_m) + J_{m-i+1}(ka\beta_m))]\}$$

The amplitudes of the electric vectors are obtained by the numerical solution of Eq. (20). In order to determine the power reflection and transmission coefficients, however, the intensities (or power flows) are necessary. By definition, the power flow in the direction of propagation of plane waves is given by the real part of the complex Poynting vector

$$\mathbf{S} = \mathbf{E} \times \mathbf{H} \tag{21}$$

From the Ponyting vector, the power reflection and power transmission coefficients can be obtained as

$$r = (E^r/E^i)^2 \tag{22}$$

and

$$t = n(\cos\theta^t/\cos\theta^i)(E^t/E^i)^2 \tag{23}$$

For normal incidence, normal reflection and transmission coefficients are

$$r = |E_o{}^r|^2 \tag{24}$$

and

$$t = n|E_o{}^t|^2 \tag{25}$$

Discussion of Results

Figure 6 illustrates the radiative properties of a rough surface with regular sinusoidal profile. The properties are given in the normal directions. In general, the profile is characterized by the expression described in Eq. (2). For regular sinusoidal profile, Eq. (2) can be reduced to $z = a \cos x/a$. Consequently, the a/λ ratio, where the wavelength of the radiation is the same as the wavelength of the roughness, is $a/\lambda = \frac{1}{2}\pi$.

This point on Fig. 6 can be considered the dividing line between specular and diffuse radiation. Furthermore, the a/λ axis can be subdivided into three regions.

Specular Region, $a/\lambda < 0.01$

The surface roughness has negligible effect on radiative properties, which can be determined by Fresnel's relations. For normal incidence Fresnel's reflectance relations can be reduced to the following expression (for both perpendicular and parallel polarization)

$$r = [(n - 1)/(n + 1)]^2 \tag{26}$$

The reflectances calculated by Fresnel's relations are shown in Fig. 6 for the refractive indices of $n = 1.5$, 2.0, 3.5 and 5.5.

Transition Region, $0.01 < a/\lambda < 1/2\pi$

The upper limit of this region corresponds to $\lambda = d = 2\pi a$ or $a/\lambda = 1/2\pi$. By following the behavior of the curve $n = 1.5$ it can be noted that above approximately $a/\lambda = 0.01$ the reflection coefficient increases, according to the energy balance of $r'xt' = 1$. Above the ratio of $a/\lambda = 0.10$ the curves of the reflection and transmission coefficients separate, giving an energy balance of less than one. The remainder of the energy propagates in other than normal direction (higher order diffracted and refracted modes). At the separation points the reflection coefficient reaches its first minimum and the transmission coefficient its maximum. The increase of transmittance was noted by Bousquet[4] already. Based on his computations, at $a/\lambda = 0.0318$ and for $n = 1.41$ the increase in transmittance is approximately 0.3%. However, this is not the maximum value. The maximum of the transmittance, as indicated by Fig. 6 is $t' = 0.985$ at $a/\lambda = 0.1$. In comparison to Fresnel's transmittance, $t = 0.96$, this is an increase of 2.5%. This finding is quite significant. First, rough surfaces can have higher transmittances than smooth ones. Second, the transmittance exhibits a maximum value at a certain a/λ ratio, which in turn is a function of the refractive index. As indicated in Fig. 6 for decreasing refractive indices, the location of the maximum transmittance approaches $a/\lambda = 1/2\pi$; i.e., $\lambda = d$. For higher refractive indices the increase in transmittance is greater. For example for $n = 5.5$ the maximum is located at $a/\lambda = 0.027$ (far below the $\lambda = d$ point) and the transmittances are $t = 0.52$ (Fresnel) and $t' = 0.62$, corresponding to an increase of about 20%.

Bousquet in his study[4] suggested a simple, closed form relation to predict the reflectances of moderately rough surfaces.

$$r' = r(1 - 2nk^2a^2) \tag{27}$$

where r is the Fresnel reflection coefficient. Figure 6 illustrates the domain of validity and the limitations of his relation around the minimum of the reflection coefficient.

Diffuse Region

This region is characterized by sudden increase in both reflectance and transmittance, although the reflectance exhibits minima and maxima of a diffraction grating. The primary objective of this study is to investigate the transition region; thus the behavior of the properties in the diffuse region will not be discussed unless it is used for comparison.

Beckmann's curve is also indicated in Fig. 6. It can be seen that in the diffuse region the reflectance obtained by the solution of the wave equation shows good qualitative agree-

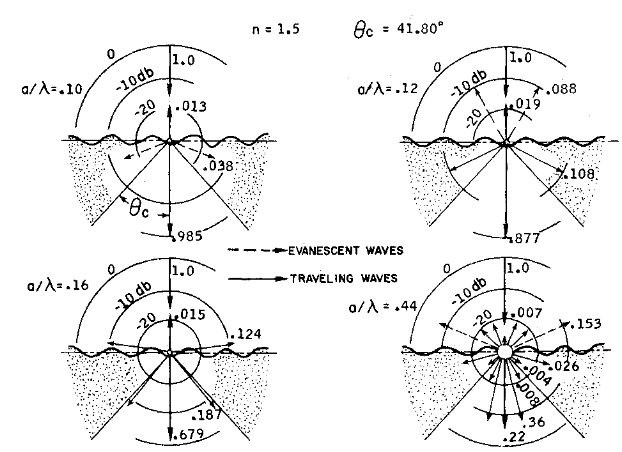

Fig. 7 Diffraction spectra for various roughnesses.

ment with Beckmann's reflectance. However, at $a/\lambda = 0.305$, where the first maximum of the reflectance occurs, the $n = 5.5$ overshoots the Beckmann curve. Furthermore, the Beckmann curve shows an earlier decrease on the a/λ axis than the one obtained by the solution of the wave equation. These deviations can be attributed to the fact that the roughness profile used for the case in Fig. 6 does not satisfy the basic assumption of Kirchhoff's method; i.e., the slope of the contour (the a/d ratio) must be small.

In the following, a few examples for the development of the diffraction spectra from specular to diffuse will be illustrated. Four cases are selected as examples: $a/\lambda = 0.10$, 0.12, 0.16 and 0.44. The refractive index is $n = 1.5$. Figure 7 shows the diffraction spectra for these cases. The intensities of the reflected and refracted beams are shown in decibels by the relation $db = 10 \log|E_o|^2$. For convenience, it is assumed that the intensity of the incident beam is unity. Furthermore, the critical angle, representing the limit of total reflection is also indicated, based on the relation $\sin\theta_c = 1/n$.

Somewhere between $a/\lambda = 0.08$ and 0.09 the first order refracted beams appear on the two sides of the zeroth order transmitted beam. Such waves do not carry energy. This can be shown by the energy balance which adds up to one (the reflectance and transmittance curves do not separate). The energy balance states that the normal components of the intensities of the total spectrum are in balance, without the consideration of evanescent waves: $|E_m{}^r|^2\alpha_m + |E_m{}^t|^2\beta_m = 1$.

Between $a/\lambda = 0.10$ and $a/\lambda = 0.11$ the first order real refracted waves appear. The energy carried away by these waves is indicated by the sudden decrease in transmittance, which can be seen in Fig. 6. At $a/\lambda = 0.12$ these waves move away from the surface. At the same time evanescent reflected waves appear. The spectrum corresponding to $a/\lambda = 0.16$ shows that as soon as the first order real transmitted waves leave the region of total reflection, real reflected waves appear on the other side of the interface close to the surface. The next case is $a/\lambda = 0.44$, showing a more diffused spectrum. It should be noted that in Fig. 6 no attempt was made to attach any physical significance to the behavior of the evanescent waves, neither to their direction of propagation nor to their amplitude.

In the preceding discussion it was assumed that the shape of the sinusoidal roughness is a regular sine curve, where the wavelength of the curve is 2π times of its amplitude. In the following the effect of the shape of the sinusoidal roughness on the surface properties will be assessed. To do this, the shape of the roughness will be normalized by introducing a shape or aspect ratio factor, f. The amplitude of the roughness will be kept as the independent variable and the pitch of the roughness, d, will be stretched by the aspect ratio, f. Then the equation of the contour can be expressed as $z = a \cos 2\pi x/df$. Thus, the new pitch is $d' = df$. For example, $f = 10$ indicates a shallow roughness where the pitch is being stretched tenfold.

Figure 8 illustrates the results of the solution of the wave equation for the case of $f = 10$. Similar to the results of Fig. 6 the Fresnel relation can be used up to approximately $a/\lambda = 0.01$. The solution is the transition region. It is even more significant that the transmittance shows a gradual decrease and no maximum is exhibited. Figure 8 also gives Beckmann's relation for comparison. The similarity between the results obtained by Rayleigh's method and Kirchhoff's method is clearly demonstrated. (For stretched roughness profiles the small d/λ ratio assumption is valid.) In order to show that Beckmann's reflectance relation is the limiting case of Rayleigh's solution, the cases of $n = 10$ and $n = 50$ are also indicated. According to these results, Rayleigh's reflectance relations with large n values are approaching rapidly the limit set by Beckmann's relation ($n \rightarrow \infty$). For very shallow contours ($a/d \rightarrow 0$) and high d/λ ratios the reflectance becomes specular again, just as for very small roughness ($a/\lambda \rightarrow 0$).

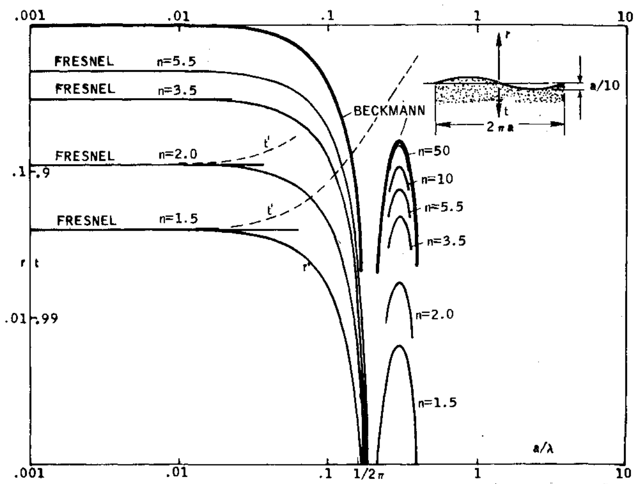

Fig. 8 Surface properties for stretched cosine contour.

However, the mechanisms are different. While very small roughnesses give rise only to zeroth order reflectance, shallow contours with high d/λ ratios may have very high number of diffracted modes but only the amplitude of the zeroth order mode is significant.

Experimental Data

Experimental work on the verification of diffraction theories is described by Stroke[3] and Pavageau.[6] Stroke's experiments were performed on sinusoidal rough surfaces with infinite conductivities. Pavageau measured reflectances over sinusoidal rough surfaces made of paraffin ($n = 1.45$), by using centimeter wave equipment. The generator used was a Klystron (2K25) and the detector a Crystal (IN23). Two wavelengths were employed, both in the X-band, 32.2 mm and 34.5 mm. The angle of incidence was varied from 0° to 60°.

A set of experiments was performed with a roughness amplitude of 2.5 mm and a pitch of 88.5 mm, corresponding to a contour aspect ratio of $f = 5.66$. For the two wavelengths of 32.2 mm and 34.5 mm the a/λ ratios were 0.0726 and 0.0724, respectively. Figure 9 shows the good agreement between theoretical predictions and experimental data.

Pavageau's second experiment was done on rough surface with 1 mm amplitude and 12 mm pitch ($f = 1.92$) at a wavelength of 34.5 mm ($a/\lambda = 0.029$). Figure 9 gives the results, indicating reasonably good agreement between theory and experiment theory: $r' = 0.0300$, experiment: $r_e' = 0.0283$; the relative error, $(r' - r_e')/r'$ is 6%. Pavageau, in his evaluation, however, reached a different conclusion. He used the Fresnel reflectance ($r = 0.0337$) in his comparison theory: $(r - r')/r = 0.11$, experiment: $(r - r')/r = 0.16$; such an evaluation gave a deviation of approximately 45%. It is felt that Pavageau's method of indirect evaluation is overly conservative. For example a 10% error by his method of evaluation would correspond to less than 1% relative error between theory and experiment.

Applications

1. Extension of the Grating Theory to Statistically Rough Surfaces

Bousquet[4] in his study proved that for normal incidence the reflection and transmission coefficients are independent of the polarization of the incident waves. He showed that for both polarizations an approximate relation can be obtained for the reflectance for small a/λ ratios, $r' = r(1 - 2nk^2a^2)$. The comparison of this approximation with the exact solution is shown in Fig. 6. Despite the narrow range of validity of this relation, it is very powerful because of its simplicity.

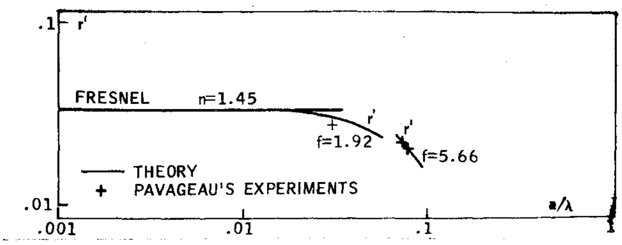

Fig. 9 Experimental data.

Bousquet extended the same relation to statistically rough surfaces, by writing the reflectance as

$$r' = r(1 - 4nk^2h^2) \tag{28}$$

where h is the roughness of the surface in rms. In view of the discussion of the influence of the aspect ratio, it can be concluded that this relation might have more constraints than the requirement for small a/λ ratio. It appears that transmittances cannot be predicted by this relation because of the strong influence of the aspect ratio.

2. Effective Properties of Rough Plates

Several investigators (Genzel,[10] McMahon,[11] Gardon,[12] Francis and Love[8]) discussed the effect of bulk properties on the radiative properties of semi-transparent materials. Their studies pointed out that the radiative properties of transparent materials are greatly influenced by the multiple internal reflections in the material. The intensity of radiation within the material is attenuated according to the absorption process. The studies in general use optically smooth surfaces for plate boundaries. In the present study, the properties of rough surfaces are used as boundary conditions. Figure 10 illustrates the physical conditions.

The propagation of energy within the plate can be described by the transport equation, which is the governing relation for the conservation of monochromatic radiant energy along a path s

$$[dI_\nu(s)]/ds = -\rho\kappa_\nu I_\nu(s) + \rho\gamma_\nu B_\nu(T) \tag{29}$$

where κ_ν is the mass absorption coefficient and γ_ν is the mass emission coefficient. By applying Kirchhoff's law, $\gamma_\nu = n^2\kappa_\nu$, and introducing the optical thickness

$$\tau = \int_0^z \rho\kappa_\nu dz \tag{30}$$

the transport equation can be written as

$$[dI_\nu(s)]/ds = -\rho\kappa_\nu[I_\nu(s) + n^2B_\nu(T)] \tag{31}$$

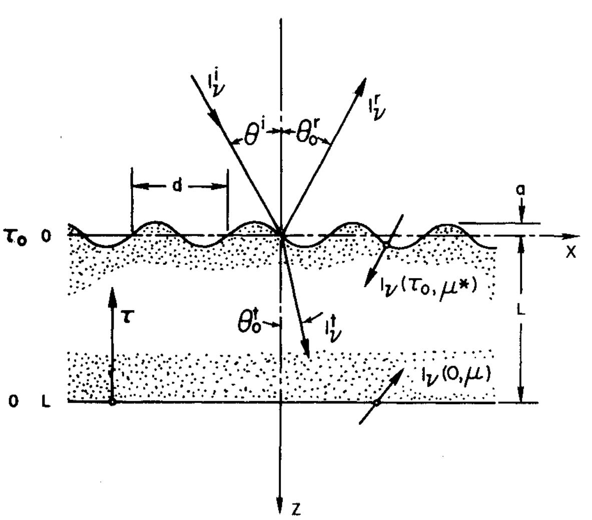

Fig. 10 Boundary conditions for the transport equation.

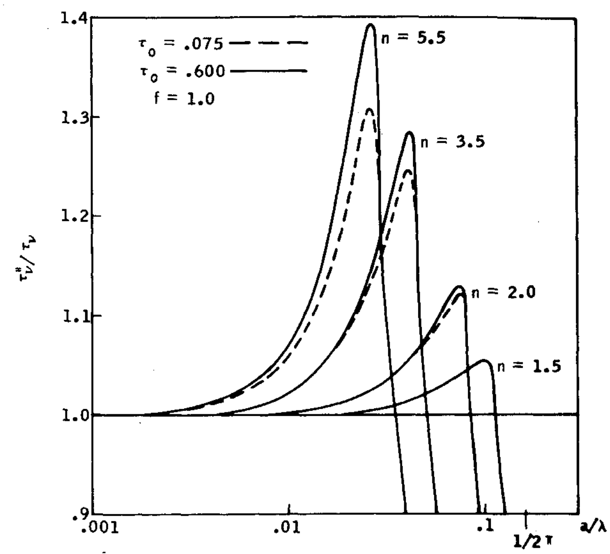

Fig. 11 Effective transmittance of a rough plate.

By defining the directional cosines of the rays as $\mu = \cos\theta$ in the z direction and $\mu^* = \cos\theta$ in the $-z$ direction, Eq. (3) can be separated into the following two equations

$$\mu(dI_\nu/d\tau) = -I_\nu(\tau,\mu) + n^2B_\nu(T) \tag{32}$$

$$-\mu(dI_\nu/d\tau) = -I_\nu(\tau,\mu^*) + n^2B_\nu(T) \tag{33}$$

Boundary conditions for these equations for plates rough on one side can be written as follows:

Intensity at the lower surface

$$I_\nu(0,\mu) = r_\nu I_\nu(0,\mu^*) \tag{34}$$

Intensity at the upper surface

$$I_\nu(\tau_0,\mu^*) = r_\nu' I_\nu(\tau_0,\mu) + t_\nu' I_\nu^i n^2 \tag{35}$$

Solution of Eqs. (32) and (33) are available in the literature for smooth surfaces, for example from the works of Heavens[13] and from the study of Francis and Love.[8]

In order to evaluate the surface properties of rough plates, the ratios of rough and smooth properties are determined for comparison. Thus the transmittance ratios are τ_ν'/τ_ν and τ_ν''/τ_ν and the emittance ratios are $\epsilon_\nu'/\epsilon_\nu$ and $\epsilon_\nu''/\epsilon_\nu$. The individual terms in these ratios are given by the following expressions.

Transmittance of smooth plate (Francis and Love)

$$\tau_\nu = (t_\nu^2 e^{-\tau_0/\mu})/(1 - r_\nu^2 e^{-2\tau_0/\mu}) \tag{36}$$

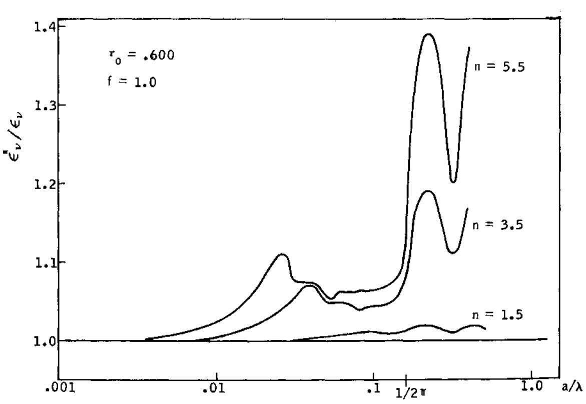

Fig. 12 Effective emittance of a rough plate.

Transmittance of a plate with rough surface on the upper side

$$\tau_\nu' = (t_\nu' t_\nu\, e^{-\tau_0/\mu})/(1 - r_\nu \tilde{r}_\nu e^{-2\tau_0/\mu}) \quad (37)$$

Transmittance of a plate with rough surface on both sides

$$\tau_\nu'' \ (t_\nu'^2 e^{-\tau_0/\mu})/(1 - r_\nu'^2 e^{-2\tau_0/\mu}) \quad (38)$$

Emittance of the smooth plate (Francis and Love)

$$\epsilon_\nu = (r_\nu - 1)(e^{-\tau_0/\mu} - 1)/(1 - r_\nu e^{-\tau_0/\mu}) \quad (39)$$

Emittance of a plate with rough surface on the upper side

$$\epsilon_\nu' = \frac{(r_\nu r_\nu' e^{-\tau_0/\mu} - r_\nu e^{-\tau_0/\mu} + r_\nu - 1)(e^{-\tau_0/\mu} - 1)}{1 - r_\nu r_\nu' e^{-2\tau_0/\mu}} \quad (40)$$

Emittance of a plate with rough surface on both sides

$$\epsilon_\nu'' = (r_\nu' - 1)(e^{-\tau_0/\mu} - 1)/(1 - r_\nu' e^{-\tau_0/\mu}) \quad (41)$$

The comparison of smooth and rough transmittance is given in Fig. 11. The increase of transmittance due to the roughness can be seen in the figure. This increase is small at low refractive indices. For example, for glasses which cover a range of approximately from $n = 1.3$ to $n = 2.05$ the increase in transmittance, at the maximum value, varies between 5 to 13%. The optical thickness has less influence on the effective transmittance. By varying the optical thickness from $\tau_0 = 0.075$ to 0.600 no change can be observed in the transmittance at $n = 1.5$. At greater refractive indices the change is more pronounced as indicated by Fig. 11. The figure also illustrates the shift of the location of the maximum transmittance as a function of the refractive index. At $a/\lambda = 0.01$ and $n = 5.5$, for example, the increase in transmittance is 7%. This corresponds to a roughness amplitude of 60 Å (reasonably smooth surface) at $\lambda = 6000$ Å.

Figure 12 illustrates the effect of roughness on the emittance. It can be seen that the increase in emittance due to the roughness for $n = 1.5$ is small, even in the diffuse region. For a refractive index of $n = 5.5$ the increase of emittance in the transition region is approximately 11%. The increase in the diffuse region is even more significant.

Conclusions

1. Summary of the Results of the Study

By representing the surface properties (reflectance and transmittance) as a function of the normalized surface roughness a/λ, it is shown that there is a transition region between optically smooth and diffuse surfaces. This region is located near the point where the pitch (or wavelength) of the roughness is the same as the wavelength of the radiation.

Within the transition region the surface transmittance may exhibit a maximum value which is greater than the one obtained by Fresnel's relation. The location of the maximum shifts towards lower a/λ ratios as the index of refraction increases.

For shallow sinusoidal contours there is no increase in transmittance in the transition region. At a refractive index of $n = 1.5$ increase in transmittance occurs only for contours deeper than approximately $f = 5$.

For roughness much smaller than the wavelength, the surface properties approach those of an optically flate plate and Fresnel's law is applicable; i.e., the radiation is specular.

Applicability and limitation of Bousquet's approximation is demonstrated for roughness much smaller than the wavelength.

If the roughness is larger than the wavelength the properties correspond to those of a diffuse plate.

In general it was found that the surface properties are influenced by both the pitch (d/λ) and the amplitude (a/λ) of the roughness in addition to the refractive index.

It is shown that for shallow roughness Beckmann's reflectance relation is the limiting case of the reflectance obtained by Rayleigh's method for the case when the refractive index approaches infinity. Furthermore, if the contour becomes more shallow; i.e., if it flattens out, the properties turn specular again.

By using the surface properties as boundary conditions, the transport equation is solved for a plate with rough boundaries. The apparent transmittance of the plate increases in the transition region. The increase is quite significant at higher refractive indices. The optical thickness has relatively small influence on the increase.

Emittance of a plate with rough surfaces is determined for various refractive indices and optical thicknesses. The influence of the optical thickness is more marked here. It should be noted that the solution for emittance is only the first approximation, considering only waves normal to the interface in the interior of the material. The complete solution would require the consideration of the contribution of waves emitted in all directions.

2. Application of the Results

a) Clarification of anomalies of surface properties of materials in all frequency bands with special consideration to optical surfaces and thin film coatings. b) Analysis of scatter and emission over rough surfaces such as ocean waves, terrestrial real surfaces and the surface of the moon. c) Change of surface and effective properties and directional behavior of materials by roughening the surface. d) Providing filtering effect by taking advantage of sharp change in properties. e) Impedance matching at microwave frequencies and suppression of the return signal by varying the roughness and the refractive index. f) Development of semiactive control system for radiative properties by changing the shape of the roughness contour, i.e., by flattening the surface.

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